Inference at $*$ 1
of proof for Lemma complete_nat_ind_with_y:

$\vdash(\lambda P,g.\ \mathrm{Y}(\lambda f,x.\ g(x,f))) \in (\forall P{:}(\mathbb{N} \to \mathbb{P}\{\mathrm{k}\}).\ (\forall i{:}\mathbb{N}.\ (\forall j{:}\mathbb{N} i.\ P(j)) \Rightarrow P(i)) \Rightarrow (\forall i{:}\mathbb{N}.\ P(i)))$
by ((MemberEqCD)
CollapseTHEN (IfLab 'subterm'
((MemberEqCD)
CollapseTHEN (
IfLab 'subterm' Id (Auto_aux (first_nat 1:n) ((first_nat 1:n),(first_nat 3:n
)) (first_tok :t) inil_term)))·
(Auto_aux (first_nat 1:n) ((first_nat 1:n
),(first_nat 3:n)) (first_tok :t) inil_term)))·

1: . . . . .subterm. T:t1:n

1. $P : \mathbb{N} \to \mathbb{P}\{\mathrm{k}\}$
2. $g : \forall i{:}\mathbb{N}.\ (\forall j{:}\mathbb{N} i.\ P(j)) \Rightarrow P(i)$
$\vdash \mathrm{Y}(\lambda f,x.\ g(x,f)) \in (\forall i{:}\mathbb{N}.\ P(i))$

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http://www.nuprl.org/FDLcontent/p0_942988_/p121_93198_{complete_nat_ind_with_y}1.html